Robust Control with Classical Methods – QFT

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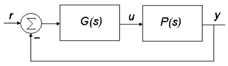
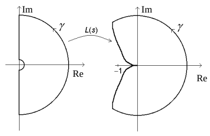
- Review of the classical Bode-Nichols control problem
- QFT in the basic Single Input Single Output (SISO) case
- Uncertainty and Fundamental Design Limitations
- QFT for non-minimum phase and computer controlled systems
- QFT for cascaded systems, and for a class of non-linear plants
- QFT for Multi-Input Multi-Output (MIMO) plants
- A comparison between QFT and other robust and adaptive control

Fundamental Design Limitations – a simplistic view

- **Stability**
  - The Nyquist stability criterion
  - Phase margin
- **Sensitivity and complementary sensitivity**
- **Bode's gain-phase relationship**
- **Bode's integral theorem**
- **Unstable poles**
- **RHP zeros and delay**
- **Exercise**
- **Example**

The Nyquist stability criterion





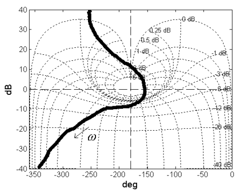
Open loop $L(s) = P(s)G(s)$

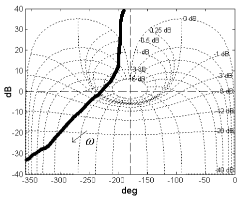
$$\frac{1}{2\pi}\underset{s\in\gamma}{\Delta \arg}(1+L(s)) = P_c - P_o$$

P_c = number of closed loop unstable poles

P_o = number of open loop unstable poles ⇒

Phase margin



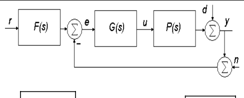


Unstable

⇒ in the cross-over frequency region: $\arg(L(j\omega_c)) \gtrsim -150°$

...

Sensitivity & complementary sensitivity



$$Y = \frac{1}{1+PG}D + \frac{FPG}{1+PG}R - \frac{PG}{1+PG}N$$

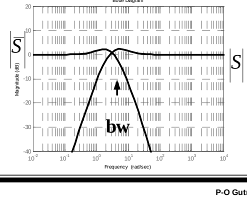
$$Y = SD + F\bar{S}R - \bar{S}N, \quad S + \bar{S} = 1$$

• Desired: $|S|$ small **and** $|\bar{S}|$ small

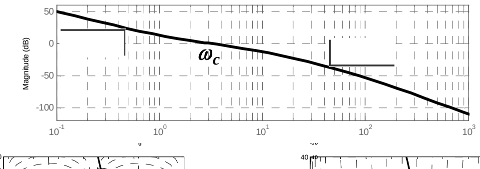
• Resolution: frequency separation

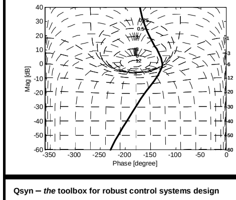
$|S|$ small for low frequencies

$|\bar{S}|$ small for high frequencies

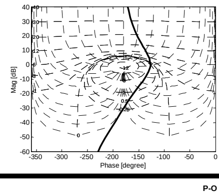


Sensitivity and complementary sensitivity





$|L(j\omega)|$



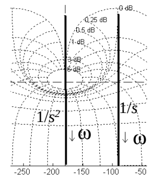
Bode's gain-phase relationship

• Assume that $L(s)$ has no poles or zeros in the RHP, and $L(s)>0$

$$\arg L(j\omega)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{d\log|L(j\zeta)|}{d\lambda}\overbrace{\log\coth\left|\frac{\lambda}{2}\right|}^{\approx\frac{\pi^2}{2}\delta(\lambda)}d\lambda \quad \text{where } \lambda=\log\left|\frac{\zeta}{\omega}\right|$$

$$\Rightarrow \arg L(j\omega)\approx\approx\frac{\pi}{2}\frac{d\log|L(j\zeta)|}{d\log\left|\frac{\zeta}{\omega}\right|}$$

$1/s$	-20 dB/dec	-90°
$1/s^2$	-40 dB/dec	-180°



• So, for desired φ_m: in cross-over frequency region, $L(s)\approx 1/s^{3/2}$ **≈-135°, -30db/dec Limited roll-off**

More about sensitivity and stability

• Recall: $S=\frac{1}{1+L},\ \bar{S}=\frac{L}{1+L}$

• Show that $S=\frac{d\bar{S}}{\bar{S}}\Big/\frac{dL}{L}$

• Of greater design interest is

$$\Delta(\bar{S})/\Delta(L)$$

• Let $L(s) = L_{nom}(s)+\Delta(s)$

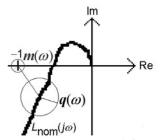
$\Delta(s)$=additive unstructured uncertainty

$|\Delta(s)| < q(s)$

• Closed loop stability margin

$\Rightarrow |1+L_{nom}(j\omega)|>q(\omega)+m(\omega)$

$$|S_{nom}(j\omega)|=\left|\frac{1}{1+L_{nom}(j\omega)}\right|<\frac{1}{q(\omega)+m(\omega)}$$



• Is it possible to make |S| arbitrarily small?

Bode's integral theorem

$$\int_0^{\infty}\ln|S(j\omega)|d\omega=\pi\sum_i p_i$$

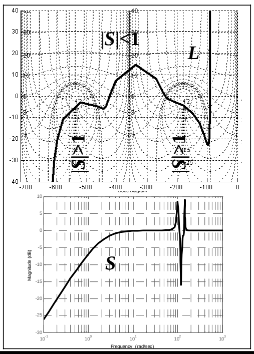
where $\{p_i\}$ are all unstable pole locations of $L(s)$.

• Water bed effect:
$\exists\omega\in(\omega_1,\omega_2)$ for which $|S(j\omega)|>0$dB

• Stable controller $G(s)$ preferrable.

• Horowitz: "optimal" sensitivity design, by double resonance/ NMP in $G(s)$. See e.g Nordin and Gutman, ECC, 1995.

• Active vibration damping.



Bode's integral theorem, cont'd

References:

•Bode
•Kwakernaak & Sivan
•Freudenberg & Looze, IEEE T-AC, 1985.
•Wu a& Jonckheere: "A simplified approach to Bode's theorem for continuous-time and discrete time systems. IEEE T-AC, 1992.
•Horowitz, ch. 10

Unstable *P*(*s*)

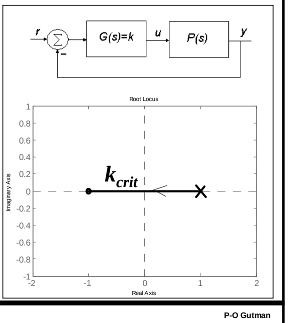
• ... sensitivity constraint.

• minimum open loop gain constraint to achieve closed loop stability:

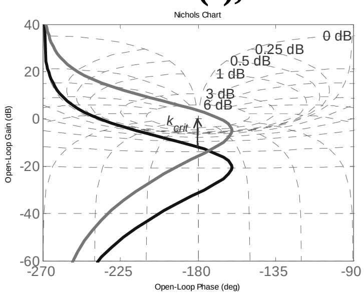
$\omega_c\geq\ \omega_{c\_crit}$ **or** $k\geq\ k_{crit}$

• Example 1: Root locus

• Example 2: Nichols chart

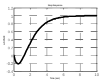


Unstable *P*(*s*), cont'd



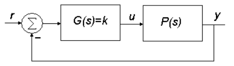
Non-minimum phase plants

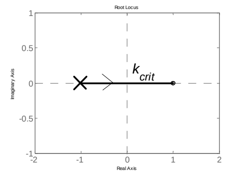
- **with RHP zero(s)**
- delay $e^{-\tau s}$
- one RHP zero step response



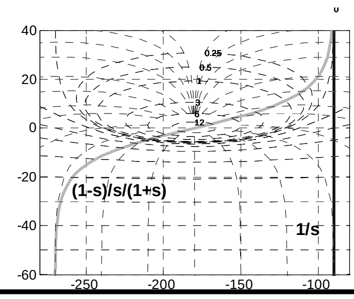
- maximum open loop gain constraint to achieve closed loop stability:

$\omega_c \leq \omega_{c\_crit}$ **or** $k \leq k_{crit}$





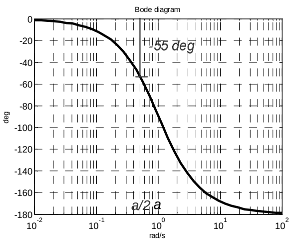
Non-minimum phase plants, cont'd



Right half plane zero

- **Let $P(s)= P_m(s)\, A(s)$,**
- **$A(s)=(a-s)/(a+s)$, $|A(s)|=1$, all-pass,**
- **$P(s)$ stable, minimum phase.**
- **Roll-off given by $G(s)\, P_m(s)$, reasonable to have around ω_c ≈ -20 dB/dec ⇒ arg ≈ -90 deg.**
- **Desired $\varphi_m \geq 35$ deg ⇒ $\arg(A(j\omega_c)) < 55$ deg ⇒**

$$\omega_c < a/2$$



Delay

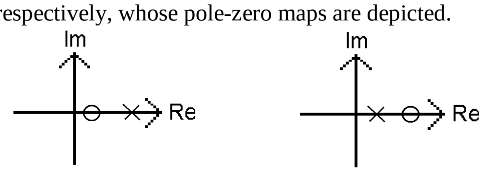
$$e^{-\tau s} \approx \frac{1-\frac{\tau}{2}s}{1+\frac{\tau}{2}s}$$

i.e. there is a RHP zero at $(2/\tau)$. Hence

$$\omega_c < 1/\tau \quad \text{[rad/s]}$$

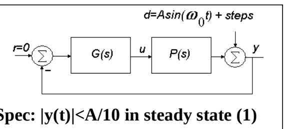
Exercise

- Find a stabilizing controller for each one of the plants, respectively, whose pole-zero maps are depicted.



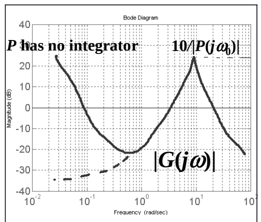
- Implications, in particular for plants with uncertainty?
- "Pole-zero interlacing property"
- "Strict stabilizability"

Example



- **Spec: |y(t)|<A/10 in steady state (1)**
  minimal control energy (2)
- **(2) ⇒ $|S(j\omega)G(j\omega)|$ as small as possible**
  ⇒ $|G(j\omega)|$ as small as possible.
- **(1) ⇒ $|S(j\omega_0)| < 0.1$ ⇒**

$$|P(j\omega_0)G(j\omega_0)| > \approx 10$$



- **Cf. active vibration control**
- **Note: not a notch filter**